

# Advances in Mind-Matter Interaction Technology: Is 100 Percent Effect Size Possible?

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**Abstract:** Very high-speed random number generators in conjunction with amplification algorithms can greatly enhance the measurements of anomalous effects and anomalous cognition. These measurements must be statistically significant and develop rapidly to become relevant and be useful in our everyday experience. Mathematical models based on a random-walk bias amplifier and experiments using GHz to THz true random bit generators hint at the possibility that measurements of mentally-influenced outputs of these generators can produce results approaching 100 percent of the corresponding intended outcomes, and at trial rates around one to two per second. Our experiments indicate feedback of results should optimally occur within about a quarter of a second of the generation of each trial so a trend may be noticeable in just a few seconds. Further, it is important the effect size be above a threshold of about 4 to 5 percent – but preferably much higher – to be psychologically “impressive.”

Key words: Mind-Matter Interaction, Anomalous Effect, Bias Amplifier, Effect Size, Random Walk.

## INTRODUCTION

Experiments intended to demonstrate the possibility mental intention can affect the measured outcome of a truly random process have been around for about 50 years<sup>(1-3)</sup>. While the statistical evidence for the validity of this effect is widespread and persuasive, the magnitude of the effect or its *effect size* has been too small to be usable<sup>(4)</sup> or even psychologically interesting to many participating subjects. After years of research to overcome these limitations we discovered a method of efficiently converting a very small effect manifesting as a bias in a large number of bits into a much larger effect in a greatly reduced number of bits – a method we call bias amplification.<sup>2</sup> In order to utilize the power of bias amplification we developed technologies enabling faster and faster true random bit generators, sometimes referred to as random event generators (REG's). The first such generators produced 16 Mbps, then an array of 64 of these generators produced an aggregate generation rate of 1 Gbps. Subsequently the rate has been steadily increased to nearly 1 Tbps in a single device.

In addition to the basic tools of bias amplification and extremely high-speed true random bit generators, a number of mathematical models have been developed to explain and quantify both the magnitude and expected behavior of the measured effects under various design conditions (References 4, 5 and in this paper). These models allowed us to put the results of various other researchers as well as our own into a context for comparison. They also provided a means of defining the apparent limits of this type of mind-matter interaction measurement and revealed some surprising possibilities.

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<sup>2</sup> The idea of increasing the size of anomalous effects or anomalous cognition using algorithmic or statistical methods has been investigated and experimented with for decades. See for example: Radin, D., 1990-91<sup>(14)</sup>, which includes an overview of earlier work by several authors.

## MATHEMATICAL MODELING

A bounded random walk is used as a bias amplifier as follows: a random walk with symmetrical bounds at plus and minus  $n$  positions from the center is incremented one step for each “1” in the input sequence and decremented for each “0.” If the bound in the positive direction is reached first, a “1” is produced at the output and the walk is reset to the center position. If the negative bound is reached first, a “0” is output and the walk is reset.

Following are the basic relationships quantifying the performance of a random walk when used as a bias amplifier. Equations 1 and 2 are adapted from solutions derived from analysis of biased bounded random walks.<sup>(6)</sup>

$$N = n \left( 1 - \left( \frac{1-p}{p} \right)^n \right) / \left( (2p-1) \left( 1 + \left( \frac{1-p}{p} \right)^n \right) \right) \quad p \neq 0.5 \quad 1a.$$

$$N = n^2 \quad p = 0.5 \quad 1b.$$

where  $N$  is the average number of steps to either boundary as a function of  $n$ , the number of positions from the starting position to a boundary, and  $p$ , the probability of a “1” occurring in the input bits; and

$$Pout = \left( 1 + \left( \frac{1-p}{p} \right)^n \right)^{-1} \quad 2.$$

where  $Pout$  is the probability of a “1” occurring in the output bits, i.e., the probability of the walk reaching the positive bound first.

The amplification factor,  $Amp$ , is defined as the output effect size,  $ES$ , divided by the input effect size:

$$Amp = \frac{2Pout - 1}{2p - 1} \quad 3.$$

Additional useful relationships may be derived from equations 1 through 3:

$$N = \frac{2Pout - 1}{2p - 1} \text{Ln} \left[ \frac{1 - Pout}{Pout} \right] / \text{Ln} \left[ \frac{1 - p}{p} \right] \quad 4.$$

giving  $N$  as a function of  $p$  and  $Pout$ .

Statistical efficiency may be defined here as the number of bits a perfectly efficient method for achieving the stated statistical result, relative to a specific method or algorithm for producing the

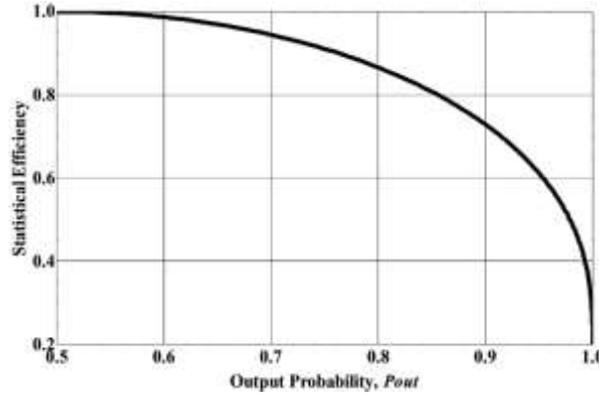
same result.<sup>(8)</sup> Statistical efficiency<sup>3</sup>,  $SE$ , is equal to the amplification factor squared divided by  $N$ :

$$SE = \frac{2Pout - 1}{2p - 1} \text{Ln} \left[ \frac{1 - p}{p} \right] / \text{Ln} \left[ \frac{1 - Pout}{Pout} \right] \quad 5.$$

For small input  $ES$  ( $-0.05 < ES < 0.05$ ) equation 5 simplifies to a function of  $Pout$  only:

$$SE \cong -2(2Pout - 1) / \text{Ln} \left[ \frac{1 - Pout}{Pout} \right] \quad 6.$$

Since the magnitude of the input  $ES$  is typically much smaller than 0.05, equation 6 can be used to plot efficiency versus  $Pout$ , which is effectively equivalent to the output hit rate,  $HR$ :



**Fig. 1**

Statistical Efficiency of the random walk bias amplifier as a function of  $Pout$ . Note, efficiency is still quite high even when the output probability (effectively the experimental hit rate) is above 85%.

By definition the average number of bits needed to compute a single output with probability  $Pout$  in a RWBA with  $N_{rw}$  input bits relative to a theoretically “perfect” bias amplifier using  $N_0$  bits is

$$N_{rw} = N_0 / SE_{rw} \quad 7.$$

From Fig. 1 a statistical efficiency of 0.8 is estimated at a hit rate of 85%. An  $SE$  of 0.8 means about 25% more bits are needed to produce a hit rate of 85% relative to a perfect bias amplifier.

The function of a random walk bias amplifier (RWBA) is effectively distributive. That means a RWBA with a bound of  $X_1$  positions followed by an RWBA of  $X_2$  positions will produce the same result as a RWBA of  $X_2$  positions followed by one of  $X_1$  positions. The same result will also be produced by a single RWBA of  $X_3 = X_1 \times X_2$  positions. These properties are vital because they allow any number of parallel generators to be combined with no loss of generality or efficiency. The only practical restriction is that all bit streams combined at any level have had equal bias amplification.

<sup>3</sup> In an earlier paper<sup>(4)</sup>  $Eff_{stat}$  is the square root of  $SE$ . The definition is changed here to provide a linear relationship with  $N$ .

A process of majority voting, sometimes called repeated guessing, means producing a single output bit from a binary input sequence based on whether there are more ones (a “majority”) or more zeros in the sequence. The number of bits in the input sequence is typically limited to odd numbers to avoid ties. Majority voting (MV) may also be considered a type of bias amplifier, but its results are not strictly distributive. For moderate  $Pout$ , reversing the order of two MV’s with the output of the first feeding bits into the second produces nearly the same final output; at high  $Pout$  this compounded MV process begins to underperform the equivalent single MV using  $N_{mv3} = N_{mv1} \times N_{mv2}$  input bits.

Majority voting is always substantially less efficient than a random walk bias amplifier, and the efficiency becomes progressively worse as MV’s are concatenated, especially at high terminal  $Pout$ . For comparison purposes the MV approach to bias amplification will be elaborated. The following equation<sup>(7)</sup> yields the exact probability,  $Pout$ , of correctly “guessing” the intended target or outcome given an input with probability,  $p$  ( $p \geq 0.5$ ), and a sequence of binary guesses (input bits) of length,  $N$ :

$$Pout = \sum_{s=a}^N p^s (1-p)^{N-s} \binom{N}{s} \quad 8.$$

where  $a = \text{Ceiling}[(N+1)/2]$ .<sup>4</sup> This equation is relatively simple, but it is only useful for fairly small  $N$  since the computation quickly becomes unwieldy. The range of the equation may be greatly extended by using logarithmic equivalents:

$$Pout = \sum_{s=a}^N \text{Exp}[s \text{Ln}[p] + (N-s) \text{Ln}[1-p] + \text{Ln}[\text{Bin}[N, s]]] \quad 9.$$

where the term  $\text{Ln}[\text{Bin}[N, s]]$  represents the natural log of the Binomial $[N, s]$ , which is calculated using a highly accurate approximation [see Appendix A]. Equation 9 extends the range of  $N$  at least up to millions, but this is still far short of the trillions necessary for a direct theoretical comparison to the performance of the RWBA. The MV process can be very accurately represented using a normal approximation to a fixed-length random walk assuming  $N$  is large:

$$Pout \cong F(\sqrt{N}(2p-1)) \quad 10.$$

where  $F(x)$  is the cumulative distribution function (CDF) of the normal distribution at  $x$ . The relative error in this approximation is less than 1% when  $N$  is as small as 21, and becomes insignificant at  $N > 100,000$ . This approximation allows the derivation of a simple equation for  $N$  as a function of  $p$  and  $Pout$ :

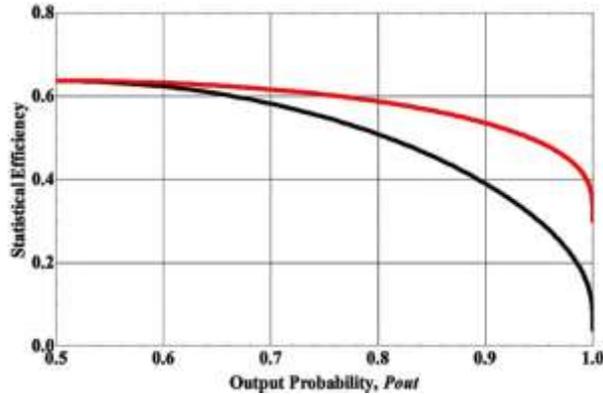
$$N \cong (F^{-1}(Pout)/(2p-1))^2 \quad 11.$$

where  $F^{-1}(y)$  is the inverse distribution function (quantile function) of the normal CDF.

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<sup>4</sup> Ceiling rounds the argument to the next higher integer.

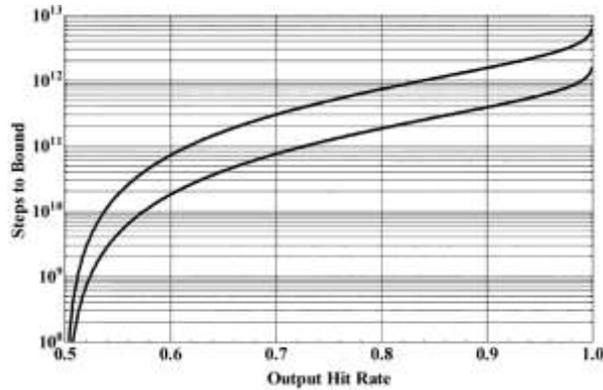
Figure 2 plots the statistical efficiency versus  $P_{out}$  (black curve) for majority voting and the relative efficiency curve (upper red) with respect to the random walk bias amplifier. It is immediately apparent the  $SE$  for the MV process is significantly less than for the RWBA. The peak  $SE_{mv}$  is  $2/\pi$  meaning at least 1.57 times the number of bits would be required by MV to accomplish a result equivalent to the RWBA. However, the relative efficiency continuously decreases as  $P_{out}$  increases. To achieve a hit rate of 99%, the majority vote process would require about 2.4 times the number of bits as a random walk bias amplifier.



**Fig. 2**

Statistical Efficiency of a majority voting process as a function of  $P_{out}$  (black curve). The top curve (red) shows the relative efficiency for majority voting versus bias amplification.

Equation 4 is used to calculate the average number of bits used to produce the specified hit rate given any input probability  $p$ .



**Fig. 3**

Figure 3 shows the average number of steps a random walker takes to reach the bound to generate the specified hit rate at the bias amplifier output. The number of steps is equivalent to the average number of random bits used in each calculation. The top curve was generated using an input  $ES$  of 0.75 ppm and the bottom curve used 1.5 ppm. These are the approximate bounds achieved for experienced operators and peak performance respectively.

Using the fact that  $\ln[(1-p)/p] \cong -2ES$ , equation 4 is simplified to the following approximation:

$$N \cong -\left( (2P_{out} - 1) \times \text{Ln} \left[ \frac{1 - P_{out}}{P_{out}} \right] \right) / 2ES^2 \quad 12.$$

where  $P_{out}$  is the output  $HR$  and  $ES$  is  $(2p-1)$ .<sup>5</sup>

For  $P_{out}$  close to 1.0, this further simplifies to

$$N \cong \frac{-\text{Ln}[1 - P_{out}]/2}{ES^2} \quad 13.$$

For  $P_{out}$  equal 0.99, Equation 13 becomes  $2.3/ES^2$  (the exact numerator is about 2.25). Equation 13 shows  $N$  increasing very slowly with increasing  $HR$  and demonstrates the apparent possibility of reaching an arbitrarily accurate mentally-intended response. Equation 13 also clearly indicates the importance of effect size of the input bits.

These predicted results are based on some critical assumptions about how an operator's conscious intention influences or interacts with true random number generators and the associated measurement and feedback system. Probably the most important assumption concerns how the effect of mental intention, commonly referred to as mind-matter interaction, enters the measurement/feedback system.<sup>(9)</sup> This subject has been debated by a number of researchers over the years, and there is still no conclusive answer. Radin (2006) published the following plot in his book, *Entangled Minds*<sup>(10)</sup> (p159, reprinted by permission from the author):

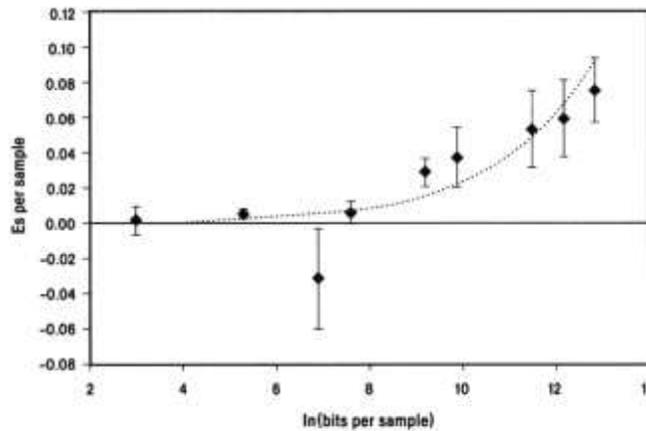


Figure 9-7. The dashed line shows the predicted increase in effect size with increasing (natural log of the) number of bits-per-button press. The diamonds with error bars show the actual results. This suggests that PK influences each bit about the same, analogous to a "mental force."

Fig. 4

The plotted data were interpreted to indicate the measured effect appeared to be a force-like or per-bit phenomenon. Assuming these data were processed by a simple majority vote algorithm and using a few data points estimated from the figure, an approximate input  $ES$  was calculated using an inverse solution to the majority voting probability. This resulted in a per-bit  $ES$  of about 180 ppm or a  $p$  of about 0.50009 for "high" intention. For comparison, data taken from an earlier

<sup>5</sup> In Wilber (2007) the *output* effect size was estimated as  $ES \approx C \cdot \sqrt{N}$ . Solving for  $N$  and comparing to equation 3, assuming statistical efficiency of 1.0, it is clear the constant,  $C$ , is the input  $ES$ ,  $2p-1$ .

publication<sup>(1)</sup> indicated an average *ES* of about 350 ppm on a per-bit basis. Other researchers, and even the same ones at different times, have proposed alternatives to the per-bit theory. One proposal is that the operator interacts with the equipment, presumably by subconscious decisions, to select the data epochs that will correspond to the intended outcome. Others propose the operator and the equipment, including the feedback or conscious “observation,” are in a type of entangled system. As such the operator and the measurement/observation are inseparable. This theory allows results to be measured forward or backward in time, as well as remotely (physically separated measurement equipment and operators).<sup>(11)</sup>

Whatever the mechanism by which mind-matter interaction becomes apparent, it is not likely to be fully explained by any of the popular theories to date.<sup>(12)</sup> Our experiments use generation rates up to almost 1 THz. Individual outputs, including feedback to the operator, are typically generated during 0.2 second intervals. At the highest generation rate, about 164 billion bits are used for each measurement. Given an *ES* of about 100 ppm estimated from published research, we should easily have seen output hit rates of virtually 100%. Even at 10 ppm input *ES*, the resulting *HR* should have been 100%. Our actual measured peak hit rates are as high as about 80% for short periods, indicating a peak input *ES* of about 1.5 ppm, or about two orders of magnitude smaller than those reported by others. This discrepancy is too large to attribute to differences in equipment or experimental setup. It has been suggested the amount of time spent in acquiring data is an important factor.<sup>(13)</sup> We collect data extremely rapidly compared to other researchers, so this is a possibly significant difference. We also use extremely high-speed generators yielding an average entropy of only about 0.3-0.5 compared to the presumably near 1.0 bit/bit of entropy provided by other researchers’ low-speed generators. At first glance the entropy difference does not seem to account for the large difference in imputed input effect size, but a deeper analysis will certainly be required.

On the other hand, we have been tracking the increase in output hit rate for about ten years as our random bit generation and processing rates have constantly increased. From the beginning the input effect size has been on the order of 1 ppm when measured using high-speed generators with cumulative generation rates of 1 GHz or faster. This represents a range in generation rates of about three orders of magnitude. Although it is not possible to determine an exact effective input *p* or output *HR*, the estimated values are nevertheless generally consistent with the bounds formed by the upper and lower curves of Figure 3.

### **Description of Hardware – PRD Systems**

The latest round of hardware development includes three levels of random bit generation rates. Each of these use Field-Programmable Gate Arrays (FPGA) as the platform for high-speed generation and data processing. The Cyclone III FPGA family is produced by Altera Corporation. These devices were selected because we have extensive experience with the Altera FPGA’s, and the Cyclone family was found to provide a good balance between speed, size, cost and ease of TRNG implementation. Tests were also done using Actel and Xilinx FPGA’s. The Actel devices were not appropriate for this application and Altera devices were selected over Xilinx due primarily to our familiarity with them.

The baseline device is called the “PsiDrive.” It uses a Cyclone III, part number EP3C10U256C8N with 10,320 logic elements (LE) to produce a combined TRNG generation rate of 6.4 GHz. This generation rate is achieved by running 32 – 200 MHz generators in parallel. Each generator includes two independent ring oscillators with multiple taps that are combined in XOR gates to produce two high-speed enhanced outputs. The enhanced outputs are sent through a series of delay lines with multiple taps and the delayed signals from each enhanced output are combined in unique pairs in

XOR gates. The combined outputs are then latched, and finally the latched outputs are combined in XOR gates into a single, raw random bit stream at 200 MHz. The raw bits are then whitened by a linear feedback shift register (LFSR) randomness corrector to produce the usable output. The corrected bits have extremely low statistical defects: less than 10-20 ppb (actual measured levels) of 1/0 bias and first-order autocorrelation. This unusually stringent requirement for statistical quality of the random sequences is necessary because the subsequent processing would amplify any stationary bias or autocorrelation resulting in biased outputs. A fundamental requirement of any psycho-responsive device (PRD) is to provide unbiased baseline data when not being influenced by mental intention.

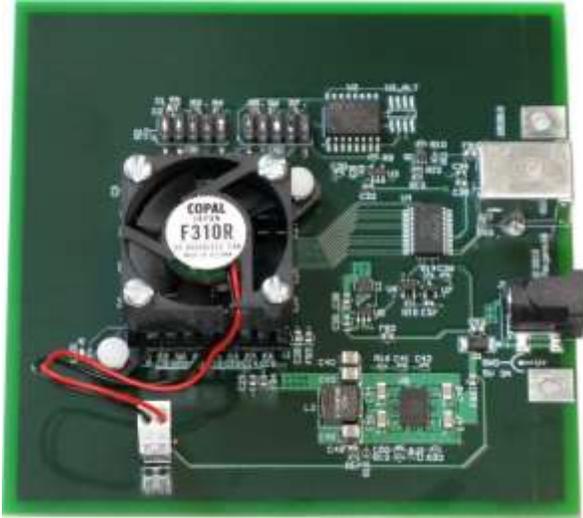
The corrected random outputs from each generator are further processed in two paths. One is the usual bias, which is a measure of the fraction of ones to total bits, and the other is autocorrelation, which is derived by converting the first-order autocorrelation into a bias contained in a converted output bit stream that is directly proportional to the autocorrelation. Each of these bit streams is passed separately through a bias amplifier and the resulting amplified streams are combined with other bit streams of the same kind. The combined streams are further amplified until the bias and autocorrelation bit streams are reduced to the desired output bit rate.



**Fig. 5** PsiDrive in Enclosure



**Fig. 6** PsiDrive PCB – 6.4 GHz



**Fig. 7** PRD 7 x32 PCB. – 204.8 GHz. Active cooling of the FPGA is required due to high power density.

The second-level device in this series, Model PRD 7 x32, is based on the largest Cyclone III FPGA, the EP3C120F484C8N with 118,088 LE to produce a combined TRNG generation rate of 204.8 GHz. The increased generation rate is accomplished using the same generator design as in the PsiDrive with 32 times the number of generators.

The third generation in the series, Model PRD 8 x128, is also based on the EP3C120F484C8N. In this case, five FPGA's are employed with four of them dedicated to generation and bit stream processing. The fifth FPGA controls and monitors the four generator IC's and combines their outputs into one bias and one autocorrelation stream, and interfaces with the USB I/O chip. The total PRD 8 generation rate is 819.2 GHz.



**Fig. 8** PRD 8 PCB – 819.2 GHz. Shown without heat sinks.

### **PRD Baseline Testing**

A large number of baseline tests were run on the PRD separately and also processed through the PsiTrainer software. The PRD hardware produces raw random bits at a rate of 891.2 GHz. This extremely high generation rate is accomplished by combining the outputs of 4096 individual generators each operating at 200 MHz. The output of each generator is passed through an LFSR whitening filter (randomness corrector), which reduces bias and first-order autocorrelation defects to less than 10 ppb. At this point each corrected generator output is used to produce two streams: the first is the unaltered stream representing the bias source, and the second is the bias source passed through a converter, which converts first-order autocorrelation into a bias in the

output equal in size to the autocorrelation. The bias and autocorrelation source streams are passed separately through several layers of bias amplification, finally resulting in two output streams at 250 Kbps each. The bias and autocorrelation of the output streams have been tested continuously up to hundreds of Gbits. One example was a test to 65.3 Gbits on each output stream. The combined raw source streams are divided by a factor of 3,276,800 in the bias amplification process so the number of raw bits tested was  $N = 2.14 \times 10^{17}$  bits. The z-scores for bias and first-order autocorrelation for both the bias and autocorrelation output streams were nominal:

Bias Stream – bias	1 <sup>st</sup> order AC	Autocorrelation Stream – bias	1 <sup>st</sup> order AC
z-score	1.02    -1.08	0.70	-1.49

The 95% confidence intervals for bias and autocorrelation with respect to the corrected source streams is:

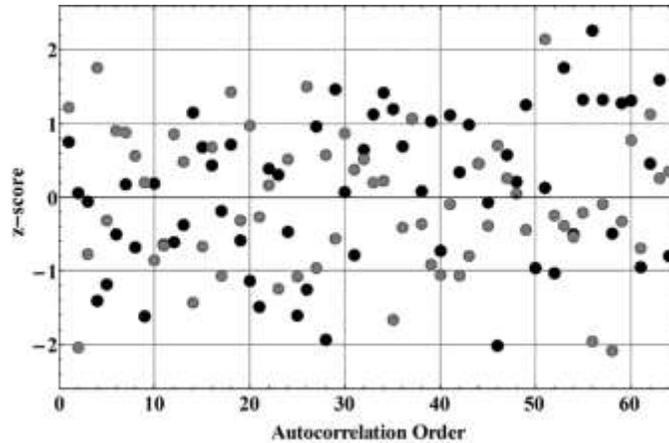
$$\pm \frac{1.96}{\sqrt{N}} = \pm 4.24 \cdot 10^{-9} \quad 14.$$

The output streams are passed through the PRDCore.dll<sup>6</sup> to the PsiTrainer where they are further processed to produce five binary bits per trial at a rate of about 25 per second, which produces one final output bit by majority-voting the five sub-trial bits. The results of the trial as well as the five sub-trial bits are subsequently stored in a data file. Majority voting is used here to average out the rather large variations in trial generation times that would result if a single output were used per trial.

Baseline testing is an automated processing and storage of the sub-trial and trial output bits. Baseline tests are expected to be unobserved, and no operator effort is required or desired. The following plot of autocorrelation was produced from a baseline series of 418,104 trials.

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<sup>6</sup> The PRDCore.dll is our software interface for various sources of random bits, including all our PRD models, a specially designed PCQNG (software-enabled PC-based TRNG) and a high quality PRNG. The dll also performs data buffering, certain processing and housekeeping tasks.



**Fig. 9**

Figure 9 show the z-scores for the first 64 autocorrelation orders of a baseline series. The series was started within 1 minute of turning on the PRD and was continued for a total of 418,104 trials. The data analyzed was the sequence of five sub-trial bits for each trial totaling 2,090,520 bits. The grey dots represent the first 250 trials (1,250 sub-trial bits) and the black dots represent the complete series. The standard deviation of each set of 64 z-scores was 0.996 and 1.014 for the first 250 and all trials respectively. Expected values are 1.0 for normally distributed z-scores.

Representative bias statistics for baseline testing (same data as used for Figure 9):

First 250 trials (1,250 sub-trial bits): mean, 0.4952; z-score, -0.339

First 250 trial bits: mean, 0.468; z-score, -1.012

All trials (2,090,520 sub-trial bits): mean, 0.5003329315; z-score, +0.963

All 418,104 trial bits: mean, 0.5002367832; z-score, +0.306

These bias and autocorrelation statistics are consistent with the assumption of unbiased and uncorrelated trials, with no indication of initial or “warm-up” effects.

## Conclusion

We provide several new mathematical tools for modeling and designing hardware devices for responding to the direct influence of mental intention. Tests over a ten-year period using devices with generation rates of one gigabit per second up to nearly one terabit per second seem to show a predictable increase in ultimate output hit rate that closely follows theoretical curves describing bias amplifiers based on bounded random walks. These theoretical equations also imply the possibility of reaching an ultimate hit rate arbitrarily close to 100 percent, although other factors relating to the psychology of a subject's belief system or our lack of understanding of the underlying mechanism involved in anomalous mental effects could certainly thwart this seemingly improbable result. Careful analysis of intermediate or sub-trial results tend to show the mental effects do not manifest strictly as a force-like or per-bit effect, but rather as a more complex combination of effects which depend on how the data is analyzed and used during a trial. This is reminiscent of quantum mechanical measurements in general, and seems to hint at why it has been so difficult to describe and reproduce anomalous mental effects in many research laboratories over the years.

Based on our empirical estimate of input bit effect size of about 1.5 ppm ( $p(1)=0.5000075$  for "High" intention), a 99 percent correct hit rate should be possible with a one terabit sample size, corresponding to a 5 THz true random generation rate and a trial duration of 200ms. This is only about a factor of six faster than our single fastest generator, so we are confident such a rate is achievable using only current technologies.<sup>7</sup> One derived equation shows the importance of input effect size on the number of bits required in each measurement, being inversely proportional to  $ES^2$ . Future research will be focused on pushing the ultimate output hit rate by a "brute force" approach of increasing generation rate to the multi-Terahertz level, along with the considerably less obvious attempts to improve input bit  $ES$ . In addition to these approaches, it seems likely there are alternative algorithms that could simultaneously address the issue of unequal trial duration of the random walk bias amplifier and remain highly statistical efficient at high hit rates.

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<sup>7</sup> At the time of this writing we are testing with an aggregate 2.5 Tbps generator. Preliminary results are consistent with those expected from the bounds indicated in Figure 3 for 500 Gbits per 200 ms trial.

## APPENDIX A

Following is a Mathematica program for calculating the natural log of the Binomial[n, k]. lnf[xx] is a routine for calculating the natural log of xx!. This function is further used in the equation, Ln[Bin[n, k]]=Ln[n!]-Ln[k!]-Ln[(n-k)!], to calculate the natural log of the desired binomial function.<sup>(15)</sup>

```
cof={76.18009172947146,-86.50532032941677,24.01409824083091,
-1.231739572450155,1.208650973866179 10^-3,-5.395239384953 10^-6};
```

```
lnf[xx_]:= (x1=xx+1.0; (*calculate Ln[xx!]*)
  If[x1<=1.,0., y=x=x1; tmp=x+5.5-(x+.5)*Log[x+5.5];
  ser=1.000000000190015; Do[(y=y+1.0; ser=ser+cof[[j+1]]/y), {j,0,5}];
  Log[2.5066282746310005*ser/x]-tmp)
```

```
lnbin[n_,k_]:= If[k==0.,0.,lnf[n]-lnf[k]-lnf[n-k]] (*calculate Ln[Binomial[n,k]]*)
```

## APPENDIX B

Additional useful equations:

$$n = \text{Ln} \left[ \frac{1 - P_{out}}{P_{out}} \right] / \text{Ln} \left[ \frac{1 - p}{p} \right] \quad 15.$$

where,  $n$ , as in equation 1, is the number of positions in the random walk required to produce  $P_{out}$  from the given  $p$ .

The drift velocity of the random walker is  $p^+ - p^-$ . That is equal to  $p - (1 - p)$ , which simplifies to  $2p - 1$ , which is equal to the  $ES$  of the input bits. Consequently, for a large  $HR$ , the number of steps to the bound converges approximately to:

$$N \cong n / ES \quad 16.$$

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